

# Aspects of $a_0$ - $f_0$ mixing in the reaction $\vec{p}n \rightarrow da_0$

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(Received 28 March 2002; published 24 July 2002)

Some aspects of  $a_0$ - $f_0$  mixing effects in the reaction  $\vec{p}n \rightarrow da_0$  with perpendicular polarized proton beam are discussed. An angular-asymmetry parameter  $A$  is defined to study those effects. It is shown that, for energies close to the production threshold, the angular-asymmetry parameter  $A(\theta, \varphi)$  is proportional to the  $a_0$ - $f_0$  mixing amplitude for arbitrary polar and azimuthal angles  $\theta$  and  $\varphi$  of the outgoing  $a_0$  meson. This statement is also valid for arbitrary energies, but then only at polar angles  $\theta=0^\circ$  and  $\theta=90^\circ$ . The mass dependence of the differential cross section  $d\sigma/dm_{\pi^0\eta}$  in the reaction  $pn \rightarrow d\pi^0\eta$  in the presence of  $a_0$ - $f_0$  mixing is also discussed.

DOI: 10.1103/PhysRevC.66.015207

PACS number(s): 13.60.Le, 13.75.-n, 14.40.Cs, 24.70.+s

## I. INTRODUCTION

The nature of the lightest, virtually mass-degenerate, scalar mesons  $a_0$  (980) ( $I^G J^{PC} = 1^- 0^{++}$ ) and  $f_0$  (980) ( $0^+ 0^{++}$ ) is an important and still unsolved problem of hadron physics. The quark structure of these mesons is not well established at present time. This issue is also closely related to the very interesting problem of  $a_0$ - $f_0$  mixing. A dynamical mechanism for this mixing close to  $K\bar{K}$  threshold was suggested around 20 years ago in Ref. [1]. Since that time a number of papers have been published in which different aspects of  $a_0$ - $f_0$  mixing and the possibilities to measure this effect have been discussed, see, for example, Refs. [2–7]. Recently, in Ref. [8], some new arguments were presented in support of a fairly large  $a_0$ - $f_0$ -mixing intensity. Analyzing the experimental data [9,10] on the exclusive production of the  $a_0$ (980),  $a_2$ (1320) and  $f_0$ (980) resonances in  $pp$  collisions at  $\sqrt{s}=29.1$  GeV, the authors of Ref. [8] came to the conclusion that  $80\% \pm 25\%$  of all  $a_0$  mesons come from the  $f_0$ (980).

The results of Ref. [8] provide certainly a fairly strong indication for a large  $a_0$ - $f_0$  mixing. However, for more solid conclusions, in particular about the quantitative value of the mixing, a thorough analysis is needed. In this context it is important to keep in mind that the mixing can manifest itself in many different physical processes and observables. Thus, in this paper we want to discuss another consequence of the  $a_0$ - $f_0$  mixing effect, namely the angular asymmetry of the reaction

$$pn \rightarrow da_0/f_0 \quad (1)$$

with unpolarized and polarized proton beam. Furthermore, we study the influence of the finite width of the scalar mesons on the unpolarized as well as the polarized differential cross section. All these investigations are performed with special emphasis on how to extract information on the  $a_0$ - $f_0$  mixing.

In the present paper we will address the  $a_0$  and  $f_0$  mesons as resonances. However, it should be stressed that all the results derived in this paper do not need any assumption

about the nature of those scalar mesons. Even if these scalar mesons are dynamically generated [11,12], in the proximity of the pole the scattering matrix can still be well approximated by the propagation of quasiparticles corresponding to elementary fields. The true nature of the propagating object is then hidden in the effective parameters of those elementary fields. This was shown rigorously by Weinberg for the case where inelastic channels are absent [13].

The paper is organized as follows. In Sec. II we review some common properties of the production of unstable particles. We start out from the case of a single particle and then, in Sec. III, generalize the formalism to the two channel case relevant for the present study. Section IV contains a discussion of the general structure of the primary production amplitude for small excess energies. Section V is devoted to the study of different observables. The main emphasis is put on the  $\pi\eta$  final state. Two differential observables are analyzed in detail, namely, the double differential cross section  $d^2\sigma/d\Omega/dm^2$  as well as the analyzing power. The former observable was studied recently in a series of papers, however, without investigating the  $m^2$  dependence more thoroughly. The latter, on the other hand, was not discussed in detail before at all. We close with a short summary and conclusions.

## II. THE PRODUCTION OF UNSTABLE PARTICLES

Let us start with the general expression for the cross section of the reaction  $NN \rightarrow dX$ , where  $X$  denotes the decay products of an unstable meson. We start the discussion by assuming the presence of one mesonic resonance only. The generalization to more than one resonance will be done in the next section. The main purpose of this section is to introduce our notation and to remind the reader on the impact of a finite decay width on observables.

The reaction cross section in the center-of-mass (c.m.) system can be expressed as

$$d\sigma_X = (2\pi)^4 \frac{1}{4|\vec{p}|\sqrt{s}} |A_X|^2 d\Phi_{\mu+1}(P; p_d, k_1, \dots, k_\mu). \quad (2)$$

Here  $\vec{p}$  denotes the c.m. momentum of the initial protons and  $P$  the total initial four-momentum. The reaction amplitude  $\mathcal{A}_X$  is given by

$$\mathcal{A}_X = W_X(k_1, \dots, k_\mu) G(m^2) \mathcal{M}, \quad (3)$$

where the primary production amplitude is denoted by  $\mathcal{M}$ . We assume the unstable meson, whose propagation is described by  $G(m^2)$ , to decay into the  $\mu$  particles of the final state  $X$  through the vertex function  $W_X$ .  $m^2$  is the total invariant mass of the final  $\mu$ -particles system and is given by  $m^2 = (\Sigma k_i)^2$ , with  $k_i$ ,  $i = 1, \dots, \mu$  being the four-momenta of the  $\mu$  decay particles. The phase space of the final  $\mu$  particles and the final deuteron (with the four-momentum  $p_d$ ) is defined as

$$\begin{aligned} d\Phi_{\mu+1}(P; p_d, k_1, \dots, k_\mu) \\ = \delta^4(P - p_d - k) \frac{d^3 p_d}{(2\pi)^3 2E_d} \prod_i \frac{d^3 k_i}{(2\pi)^3 2\omega_i} \\ = (2\pi)^3 dm^2 d\Phi_2(P; p_d, k) d\Phi_\mu(k; k_1, \dots, k_\mu), \end{aligned} \quad (4)$$

where  $E_d$  and  $\omega_i$  are the energies of the final deuteron and the  $i$ th decay particle, respectively, and  $k = \Sigma k_i$ . The latter recursive formula allows us to study the propagation and the decay of the unstable particles independently of the production mechanism itself.

It is convenient to introduce what we would like to call partial spectral functions  $\rho_X$ , which is given by

$$\rho_X(m^2) := (2\pi)^3 \int d\Phi_\mu(k; k_1, \dots, k_\mu) |G(m^2) W_X|^2. \quad (5)$$

Note that unitarity demands that

$$\sum_X \rho_X(m^2) = -\frac{1}{\pi} \text{Im} G(m^2), \quad \int dm^2 \sum_X \rho_X(m^2) = 1, \quad (6)$$

where the sum runs over all decay channels of the unstable particle that are open at a certain given value of  $m^2$ .

Using Eqs. (2)–(5) we can write

$$\frac{d^2 \sigma_X}{dm^2 d\Omega_k} = \frac{1}{64\pi^2} \left( \frac{|\vec{k}|}{|\vec{p}|} \right) \frac{1}{s} |\mathcal{M}|^2 \rho_X(m^2), \quad (7)$$

where  $\vec{k}$  is the relative momentum of the meson system  $X$  with respect to the deuteron. Naturally,  $|\vec{k}|$  is a function of  $m^2$ , namely,

$$|\vec{k}| = \frac{\lambda^{1/2}(s, m^2, M_d^2)}{2\sqrt{s}}, \quad (8)$$

where  $M_d$  denotes the mass of the deuteron and the function  $\lambda$  is given by

$$\lambda(x, y, z) = (x - y - z)^2 - 4yz. \quad (9)$$

$$\text{thin line} = \text{thick line} + \text{thick line} \text{---} \text{circle } \Sigma \text{---} \text{thick line}; \quad \text{circle } \Sigma = \sum_X \left( \text{loop diagram} \right)_X$$

FIG. 1. Graphical representation of the Dyson equation. The thin solid line denotes the bare propagator whereas the thick solid line stands for the dressed/physical propagator. The sum appearing in the expression for the self-energy is assumed to run over all decay channels  $X$ .

In this paper we neglect a possible final state interaction (FSI) of the produced mesons with the deuteron (this issue was discussed for the reaction  $pp \rightarrow da_0^+$  in Ref. [14]). Then, close to threshold, the energy dependence of the matrix element  $\mathcal{M}$  is determined by the lowest possible orbital angular momentum  $l$  of the meson system  $X$  with respect to the deuteron. Specifically, it will be given by  $k^l$  with  $k = |\vec{k}|$  being the corresponding relative momentum.

Let us now consider the case that interests us in the present study, namely the decay of a scalar resonance or quasiparticle into two pseudoscalar mesons. For any given interaction of the scalar mesons with the pseudoscalars, the physical two point functions for the scalar mesons can then be constructed by solving the Dyson equation, where the self-energy  $\Sigma$  is given by the leading order two pseudoscalar loop diagrams. The equation is illustrated in Fig. 1. The strongest energy dependence of the loops is introduced by the unitarity cut and its analytical continuation below the threshold. The remaining piece can be assumed constant and can be absorbed in the physical mass  $M_R$ . Under these assumptions the physical propagators of the scalar mesons can be described by a Flatté form [15], namely by

$$G(m^2) = \frac{1}{m^2 - M_R^2 + iM_R \sum_X \Gamma_X}, \quad (10)$$

where

$$\Gamma_X = \frac{|W_X|^2 |\vec{k}_X|}{8\pi M_R^2},$$

with  $\vec{k}_X$  being the c.m. momentum of the meson system  $X$ . Below the threshold of the production of the final state  $X$  the analytic continuation of  $\Gamma_X$  is to be used.

### III. THE MIXING AMPLITUDE

We now turn to the case that is relevant for the present paper, namely when there are two propagating unstable particles with different isospins. Then the self-energy  $\Sigma$  exhibits a matrix structure in isospin space. If we now allow the propagating particles to mix, then the self-energy matrix develops nondiagonal elements. Note, since the mixing particles  $a_0$  and  $f_0$  are essentially mass degenerate, all the mixing should be completely dominated by its effect on the meson propagation itself. Along the lines of this reasoning we completely neglect any isospin violation in the primary production amplitude  $\mathcal{M}$  and in the coupling of the pseudo-

scalar mesons to the resonances. In short, we assume the resonances to be produced in pure isospin states. The physical states, however, are those that propagate. The physical states of  $a_0$  and  $f_0$  are consequently no longer pure isospin states.

We start from the inverse propagator in the isospin basis,

$$\mathbf{G}^{-1}(m^2) = \begin{pmatrix} (m^2 - m_1^2 - \Sigma_{11}) & -\Sigma_{10} \\ -\Sigma_{01} & (m^2 - m_0^2 - \Sigma_{00}) \end{pmatrix} \\ =: \begin{pmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{pmatrix},$$

where  $m_1$  and  $m_0$  denote the bare masses in the isospin 1 and 0 channel, respectively. The diagonal elements of the self-energy are given by

$$\Sigma_{11} = \Pi_{\bar{K}K}^1 + \Pi_{\pi\eta}, \quad (11)$$

$$\Sigma_{00} = \Pi_{\bar{K}K}^0 + \Pi_{\pi\pi}, \quad (12)$$

and the isospin breaking is generated by

$$\Sigma_{10} = \Sigma_{01} = \alpha + \Pi_{K^+K^-} - \Pi_{\bar{K}^0K^0}. \quad (13)$$

In these formulas  $\Pi_{xy}$  denote loops integrals with particles  $x$  and  $y$  propagating. The loops are assumed to be renormalized—the renormalization constants are either absorbed into the physical masses of the scalar mesons or into the complex quantity  $\alpha$  (see also the discussion of this question in Ref. [16]). The loops denoted by  $\Pi_{\bar{K}K}^I$  are to be read as containing contributions from charged kaons as well as neutral kaons, coupled to the isospin  $I$  as indicated in the superscript. It is also implied that the physical masses are used in those loop integrals. The mass difference between the charged and the neutral kaons is the source for the isospin breaking generated by the loops of Eq. (13). Without this difference the two kaon loops would cancel each other. The complex quantity  $\alpha$  contains the effect induced by  $\pi$ - $\eta$  mixing [17], as for instance discussed in Ref. [6], as well as a possibly existing direct  $a_0$ - $f_0$  transition. Naturally, the imaginary part of  $\alpha$ , induced by the  $\pi^0$ - $\eta$  transition, can be easily estimated as was done in Ref. [6].

The propagation of the physical states is described by the eigenvalues of  $\mathbf{G}$ , i.e.,

$$G_{a_0} = 2/(g_{11} + g_{22} + r) \quad \text{and} \quad G_{f_0} = 2/(g_{11} + g_{22} - r), \quad (14)$$

where  $r = \sqrt{(g_{11} - g_{22})^2 + 4g_{12}^2}$ . The physical masses of the scalar mesons  $a_0$  and  $f_0$  are given by the real part of the zeros of Eq. (14).

As usual the full propagator  $\mathbf{G}$  can be expressed in terms of the physical propagators  $G_{a_0}$  and  $G_{f_0}$  plus a parameter for the mixing, which can be identified as  $\delta = 2g_{12}/(g_{11} - g_{22})$ . In the case of  $\rho$ - $\omega$  mixing or  $\pi$ - $\eta$  mixing  $\delta$  is guaranteed to be small due to a large denominator  $g_{11} - g_{22}$ . In the former case there is a large difference in the widths whereas in the latter the masses of the mixing particles are rather different.

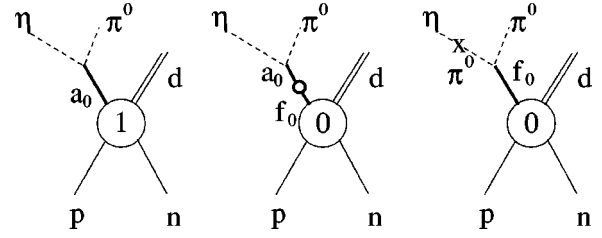


FIG. 2. Different contributions to  $\pi\eta$  production in leading order in the mixing. Here  $f_0$ - $a_0$  mixing is denoted by a circle and  $\pi$ - $\eta$  mixing is denoted by a cross. Note that  $f_0$ - $a_0$  mixing can occur via  $K\bar{K}$  loops as well as via direct mixing, as explained in Sec. III. The numbers in the primary production vertices (denoted by a big circle) indicate the isospin of the relevant amplitude.

However, the experimental evidence that is available for the masses and widths of the scalar mesons  $a_0$  and  $f_0$  suggests that here both these quantities might be very similar [18]. Thus, even  $g_{11} = g_{22}$  is not excluded. Also, we want to emphasize that  $\delta$  cannot be interpreted as a mixing angle because it is necessarily complex valued.

For convenience we write the primary production amplitude  $\mathcal{M}$  as a vector in isospin space,

$$\mathbf{M} = \begin{pmatrix} \mathcal{M}_1 \\ \mathcal{M}_0 \end{pmatrix},$$

and the final production vertex  $W_X$  as a matrix

$$\mathbf{W} = \begin{pmatrix} W_1^{(\pi\eta)} & W_1^{(\pi\pi)} \\ W_0^{(\pi\eta)} & W_0^{(\pi\pi)} \end{pmatrix}.$$

Note that here  $W_1^{(\pi\eta)}$  is the vertex for the coupling of the  $\pi\eta$  system to an isospin-one particle as given by an elementary Lagrangian, whereas  $W_0^{(\pi\eta)}$  is the vertex where the  $\pi\eta$  system mixes first into the  $\pi\pi$  system which then couples to an isospin-zero particle. We thus find for the complete production amplitude of the final states  $X = (\eta\pi)$  or  $(\pi\pi)$  [cf. Eq. (3) for the one channel situation]

$$\mathbf{A} = \mathbf{W}^\dagger \mathbf{G} \mathbf{M} = \mathbf{W}^\dagger \frac{1}{g_{11}g_{22} - g_{12}^2} \begin{pmatrix} g_{22} & -g_{12} \\ -g_{12} & g_{11} \end{pmatrix} \mathbf{M} \\ = \mathbf{W}^\dagger G_{a_0} G_{f_0} \begin{pmatrix} g_{22} & -g_{12} \\ -g_{12} & g_{11} \end{pmatrix} \mathbf{M}.$$

Basically all investigations on the  $a_0$ - $f_0$  so far do not use the propagators of the physical states, i.e., they do not use the eigenvalues of  $\mathbf{G}$ . Rather they assume that the  $a_0$  and  $f_0$  mesons still propagate as proper isospin states. This leads to expressions that are very easy to interpret and which are illustrated in Fig. 2 for the case of  $X = (\pi\eta)$ . Within the formalism developed above this assumption corresponds to the approximation

$$G_{f_0} G_{a_0} = \frac{1}{g_{11}g_{22} - g_{12}^2} \simeq \frac{1}{g_{11}g_{22}} = \tilde{G}_{f_0} \tilde{G}_{a_0}, \quad (15)$$

where  $\tilde{G}_{f_0}$  and  $\tilde{G}_{a_0}$  are now propagators of these mesons in the isospin 0 and 1 states, respectively. Indeed, the results presented in Ref. [6] suggest that

$$\frac{g_{12}^2}{g_{11}g_{22}} < 10^{-2},$$

i.e., that the approximation (15) is not unreasonable.

Since the main focus of the paper is the  $\pi\eta$  channel it is convenient to introduce a mixing parameter  $\xi$  by

$$\xi(m) = \Sigma_{01}(m) \tilde{G}_{f_0}(m^2), \quad (16)$$

where  $\Sigma_{01}$  was defined in Eq. (13) and is related to the  $a_0^0 f_0$  transition amplitude. With this definition we get for the transition amplitude for the reaction  $pn \rightarrow d\pi\eta$

$$\begin{aligned} \mathcal{A}_{(\pi\eta)} = & W_1^{(\pi\eta)} \tilde{G}_{a_0} (\mathcal{M}_1 + \xi(m) \mathcal{M}_0) + W_0^{(\pi\eta)} \tilde{G}_{f_0} \mathcal{M}_0 \\ & + W_0^{(\pi\eta)} \xi(m) \tilde{G}_{a_0} \mathcal{M}_1. \end{aligned} \quad (17)$$

The last term involves mixing in  $W_0^{(\pi\eta)}$  as well as in  $\xi(m)$  and is therefore a second order correction. Close inspection reveals that in the region  $m \approx 2m_K$  the third term in Eq. (17) should be suppressed by a factor

$$\frac{g_{f_0\pi\pi} |\lambda_{\pi^0\eta}| m_{a_0} \Gamma_{a_0}}{g_{a_0\pi\eta} \Sigma_{01}(m_\eta^2 - m_\pi^2)} \approx 0.12$$

compared to the second term. In this estimation we used the values  $\Sigma_{01} \approx 5000 \text{ MeV}^2$  determined from  $K\bar{K}$  decay loops (see Fig. 2 in Ref. [6]) at  $m \approx 2m_K$  and the  $\pi\eta$  mixing amplitude  $\lambda_{\pi^0\eta} \approx -5000 \text{ MeV}^2$  (see Ref. [6] and references therein). We also use the masses and widths  $m_{a_0} = m_{f_0} = 980 \text{ MeV}/c^2$  and  $\Gamma_{a_0} = \Gamma_{f_0} = 50 \text{ MeV}/c^2$  and coupling constants  $g_{a_0\pi\eta}$  and  $g_{f_0\pi\pi}$  that are determined from the equations  $\Gamma_{a_0} = g_{a_0\pi\eta}^2 q_{\pi\eta} / (8\pi m_{a_0}^2)$  and  $\Gamma_{f_0} = 3g_{f_0\pi\pi}^2 q_{\pi\pi} / (16\pi m_{f_0}^2)$ , where  $q_{\pi\eta}$  and  $q_{\pi\pi}$  are corresponding relative momenta. Consequently, both terms—the third and the last in Eq. (17)—will be neglected in what follows.

#### IV. STRUCTURE OF THE PRIMARY PRODUCTION AMPLITUDE

As we argued in the former section, one can safely assume the primary production amplitudes  $\mathcal{M}_I$  as isospin conserving. In this section we demonstrate how to construct the effective interaction relevant for the transitions  $pn \rightarrow d + (\text{scalar meson})$  in the close-to-threshold region.

As pointed out in Refs. [6,19], if the scalar meson has isospin 1 it can only be produced in a  $p$  wave. Since the initial  $pn$  system has to be in an isotriplet state and has to have odd parity, the Pauli principle requires that it is also in a spin-triplet state. Thus, the effective transition operator for the isovector final state has to be linear in  $\mathbf{k}$  and has to have

an odd power of  $\mathbf{p}$  (hereafter  $\mathbf{k}$  and  $\mathbf{p}$  denote the final and initial relative three-momenta in the considered reaction, respectively). It also has to be linear in both the spin  $\mathbf{S} := \phi_1^T \boldsymbol{\sigma}_2 \boldsymbol{\sigma} \phi_2$  of the initial nucleons pair and the polarization vector  $\boldsymbol{\epsilon}$  of the outgoing deuteron. These constraints lead to the reaction amplitude of the following type:

$$\begin{aligned} \mathcal{M}_1 = & a(\mathbf{p} \cdot \mathbf{S})(\mathbf{k} \cdot \boldsymbol{\epsilon}^*) + b(\mathbf{p} \cdot \mathbf{k})(\mathbf{S} \cdot \boldsymbol{\epsilon}^*) + c(\mathbf{k} \cdot \mathbf{S})(\mathbf{p} \cdot \boldsymbol{\epsilon}^*) \\ & + d(\mathbf{p} \cdot \mathbf{S})(\mathbf{p} \cdot \boldsymbol{\epsilon}^*)(\mathbf{k} \cdot \mathbf{p}). \end{aligned} \quad (18)$$

The coefficients  $a$ ,  $b$ ,  $c$  and  $d$  are independent scalar amplitudes. They may depend on the total c.m. energy and are necessarily complex since they contain the initial state interaction. For the isospin 1 amplitude relevant here the phase induced by the two nucleon unitarity cuts can be related to the  $NN$  phase shifts, as pointed out in Ref. [20]. However, for the isoscalar initial state (required for the production of the  $f_0$  meson, cf. below) there is no phase-shift analysis available at the relevant energies due to a lack of high energy  $pn$  scattering experiments [21]. Therefore, in this paper we will not consider the effects induced by the initial state interaction. Let us mention, however, that their effects do not influence the qualitative aspects discussed here. Note that the scalar amplitudes  $a$ ,  $b$ ,  $c$ , and  $d$  are expected to have quite smooth energy dependence and, accordingly, can be considered as constant in the near threshold region.

In Ref. [14] the transition amplitude was given in a partial wave decomposed form. Obviously the two descriptions are equivalent, however, we regard Eq. (18) as more convenient for the construction of polarization observables.

The amplitude relevant for the near-threshold production of an isoscalar scalar meson can be constructed along the same lines and is given by

$$\mathcal{M}_0 = f(\mathbf{S} \cdot \boldsymbol{\epsilon}^*) + g(\mathbf{p} \cdot \mathbf{S})(\mathbf{p} \cdot \boldsymbol{\epsilon}^*). \quad (19)$$

Similar expressions for the amplitudes (18) and (19) can be found in Refs. [7,22]. Note, however, that the terms proportional to  $g$  and  $d$  were omitted in that work.

It is obvious from Eqs. (18) and (19) that  $\mathcal{M}_1 \propto k$  and  $\mathcal{M}_0 \propto \text{const}$  near the threshold. For the isospin conserving case we therefore get

$$\frac{d\sigma}{dm^2}(pn \rightarrow da_0) \propto Q^{3/2} \quad \text{and} \quad \frac{d\sigma}{dm^2}(pn \rightarrow df_0) \propto Q^{1/2},$$

where  $Q(m) = \sqrt{s} - M_d - m$  is the excess energy. Thus, if we study the production of an isospin 1 final state, namely  $\pi^0\eta$ , the mixing with the isoscalar ( $f_0$ ) state will be kinematically enhanced [6].

#### V. STUDY OF OBSERVABLES

##### A. Invariant mass spectrum

Naturally the simplest observable is just the invariant mass spectrum of the reaction  $pn \rightarrow dX$ . As long as we look at energies/values of  $m^2$  such that  $Q(m^2)$  is small, the spectrum should show a resonant peak  $\sim \Sigma_X \rho_X$  reflecting the  $m^2$



dependence of the  $f_0$  propagator, cf. Eq. (6). It is possible to deduce the branching ratios of a particular resonance by a fit of the Flatté distribution to the mass spectrum. This was demonstrated, e.g., for the case of the  $a_0^+$  in Ref. [23]. Since so far little is known about the  $f_0$  this very easy to measure observable is very interesting. In addition, the number of all events in this peak is proportional to

$$|\mathcal{M}_0|^2 = 3|f|^2 + 2\operatorname{Re}(fg^*)p^2 + |g|^2p^4,$$

cf. Eqs. (6), (7) and (19). Note that the knowledge of the amplitudes  $f$  and  $g$  is important, if we want to deduce quantitative informations about the  $f_0/a_0$  mixing amplitude from the reaction  $pn \rightarrow dX$ .

Definitely the most interesting observables are those concerning the channel  $pn \rightarrow d\pi\eta$ , since here the isospin conserving amplitude enters in a  $p$  wave whereas the isoscalar amplitude, which can be mixed in via isospin violation, enters in an  $s$  wave. This is the kinematical enhancement mentioned at the end of the last section.

Let us discuss the mass distribution  $d\sigma/dm_{\pi^0\eta}$  for the  $\pi^0\eta$  system in the reaction  $pn \rightarrow d\pi^0\eta$ . Note that near the threshold of the reaction  $pn \rightarrow da_0^0$  this mass distribution should be sensitive to the magnitude of  $a_0^0$ - $f_0$  mixing as well as to the mixing mechanism.

Consider first the limiting case of isospin conservation, so that the  $a_0^0$ - $f_0$  mixing is absent. In this case the mass spectrum of the  $\pi^0\eta$  system should coincide with the spectrum of the  $\pi^+\eta$  system from the reaction  $pp \rightarrow da_0^+$ —apart from an overall (isospin) factor of 0.5 [7]. The amplitude of the reaction  $pn \rightarrow d\pi^0\eta$  is then given by the diagram of Fig. 2(a). In this case the invariant  $\pi^0\eta$ -mass distribution is determined by the phase space and  $\rho_{(\pi\eta)}$ , which was defined in Eq. (5), and is dominated by the  $a_0$  propagator [cf. Eq. (10)]. The  $\pi^0\eta$ -mass distribution obtained under these assumptions is shown in Fig. 3 by the dotted line. The calculations are done at  $T_p = 2645$  MeV/c where experimental results can be expected from the ANKE Collaboration soon [22]. The parameters used here are  $M_{a_0} = 980$  MeV/ $c^2$  and  $\Gamma_{a_0} = 50$  MeV/ $c^2$ . The mass-dependent width is described by a Flatté-like form  $\Gamma_{a_0}(m) = \Gamma_{a_0} + \Gamma_{K\bar{K}}(m)$ , which takes into account the  $K\bar{K}$  decay channel [see Eq. (8) in Ref. [6]]. As we discussed in the previous sections, the isospin conserving production of the  $a_0^0$  meson can take place only in a  $p$  wave and hence is suppressed near threshold.

If isospin is not conserved and  $a_0^0$ - $f_0$  mixing is present, then the  $\pi\eta$  system can be produced also in an  $s$  wave through the  $f_0$ . The corresponding diagram is shown in Fig. 2(b). For this mechanism the  $\pi^0\eta$  mass spectrum is determined not only by the  $a_0^0$  propagator, the partial wave of production and the phase space but also by the  $f_0$  propagator and by the mass dependence of the nondiagonal self-energy matrix element  $\Sigma_{01}(m)$  [cf. Eqs. (13) and (16)]:

$$\xi(m) = \Sigma_{01}(m) \tilde{G}_{f_0}(m^2) \approx \frac{\Sigma_{01}(m)}{m^2 - M_{f_0}^2 + iM_{f_0}\Gamma_{f_0}(m^2)}.$$

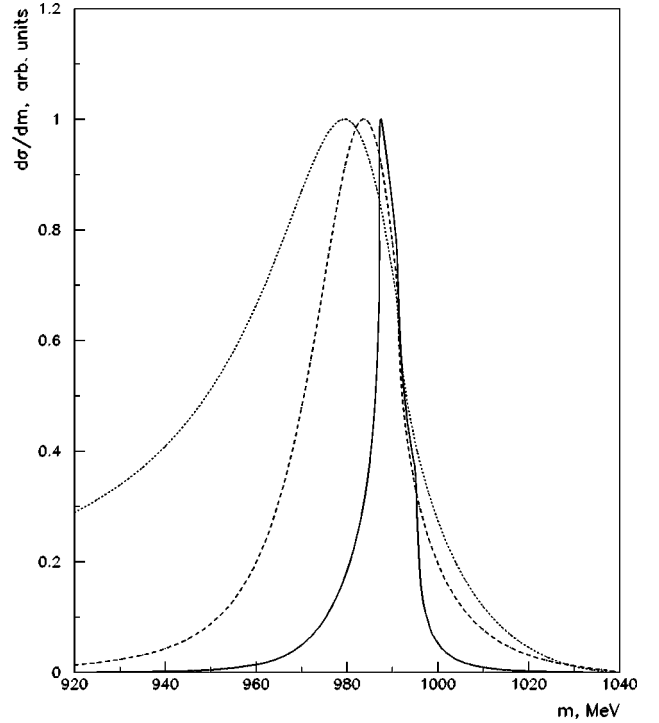


FIG. 3. Spectra of the  $\pi^0\eta$  invariant mass for the reaction  $pn \rightarrow d\pi^0\eta$  at  $T_p = 2645$  MeV/c. The dotted line corresponds to a reaction mechanism where an  $a_0$  meson is produced in a  $p$ -wave and then decays into the  $\pi^0\eta$  system. The dashed line corresponds to the production of an  $f_0$  meson in an  $s$ -wave that mixes into an  $a_0$  with  $\Sigma_{10} = \text{const}$ , cf. Eq. (13). The solid line shows results where  $\Sigma_{10}$  is evaluated from the  $K\bar{K}$  mixing mechanism. Note that all the distributions are normalized to 1 at their maximal value.

For illustrative purposes we will use  $M_{f_0} = 980$  MeV/ $c^2$  and the same Flatté-like width that was used for the  $a_0$  meson above, i.e.,  $\Gamma_{f_0}(m^2) = \Gamma_{a_0}(m^2)$ .

Of course, now the invariant mass plot becomes sensitive to the specific mixing mechanism. For example, if we assume that the leading contribution to  $\Sigma_{01}(m)$  comes from the direct  $a_0^0$ - $f_0$  transition mechanism, i.e., the term  $\alpha$  in Eq. (13), then there should be no drastic dependence on the invariant mass  $m$  of the  $\pi^0\eta$  system. The  $\pi^0\eta$ -mass spectrum, corresponding to the mechanism of Fig. 2(b) and with  $\Sigma_{01}(m) = \text{const}$ , is shown in Fig. 3 by the dashed line. (All the distributions are normalized to 1 at the maximal values.) On the other hand, if  $\Sigma_{01}(m)$  is determined mainly by the  $K\bar{K}$  loop [the term  $\Pi_{K^+K^-} - \Pi_{\bar{K}^0K^0}$  in Eq. (13)], it should be strongly enhanced near the  $K\bar{K}$  threshold (see, e.g., Refs. [1,4,6]). The corresponding  $\pi^0\eta$ -mass distribution is shown in Fig. 3 by the solid line.

Thus, we conclude that the  $\pi^0\eta$  mass spectrum is rather sensitive to the  $a_0^0$ - $f_0$  mixing mechanism. That is why the study of  $d\sigma/dm_{\pi^0\eta}$  in the reaction  $pn \rightarrow d\pi^0\eta$  should allow us to shed light on the nature of the lightest scalar mesons  $a_0(980)$  and  $f_0(980)$ .

## B. Analyzing power for the process $pn \rightarrow d\pi\eta$

### 1. Near threshold

In our approximation the total amplitude  $\mathcal{M}$  of the resonant  $\pi\eta$  production with  $a_0^0$ - $f_0$  mixing effects taken into account may be written as [cf. Eq. (17)]

$$\mathcal{M} = \mathcal{M}_1 + \xi \mathcal{M}_0, \quad (20)$$

where the amplitudes  $\mathcal{M}_1$  and  $\mathcal{M}_0$  are given by the Eqs. (18) and (19) and the mixing parameter  $\xi$  was defined in Eq. (16).

Let us introduce the polarization of one of the nucleons  $\zeta$  through  $\zeta = \phi_1^+ \sigma \phi_1$  or  $\phi_1 \phi_1^+ = (1 + \zeta \cdot \sigma)/2$ . The matrix element (20) squared and averaged (summed) over the polarizations of the initial neutron (final deuteron) is then given by

$$\begin{aligned} \overline{|\mathcal{M}|^2} = & \frac{1}{2} [(|a|^2 + |c|^2)p^2k^2 + 3|B|^2 + |D|^2p^4] + (\mathbf{p} \cdot \mathbf{k}) \text{Re} H \\ & + p^2 \text{Re}[a^* D(\mathbf{p} \cdot \mathbf{k}) + B^* D] + (\zeta \cdot [\mathbf{k} \times \mathbf{p}]) \text{Im} H, \end{aligned} \quad (21)$$

where

$$B = b(\mathbf{p} \cdot \mathbf{k}) + f\xi, \quad D = d(\mathbf{p} \cdot \mathbf{k}) + g\xi,$$

$$H = a^* B + a^* c(\mathbf{p} \cdot \mathbf{k}) + B^* c + D^* c p^2.$$

Let the polarization vector  $\zeta$  be directed along the  $x$  axis ( $\zeta \perp \mathbf{p}$ ). Then, in terms of the angles  $\theta$  and  $\varphi$ , we have  $\mathbf{p} \cdot \mathbf{k} = pk \cos \theta$  and  $\zeta \cdot [\mathbf{k} \times \mathbf{p}] = \zeta p k \sin \theta \sin \varphi$  ( $\zeta = |\zeta|$ ). Using those expressions Eq. (21) may be written as

$$\begin{aligned} \overline{|\mathcal{M}(\theta, \varphi)|^2} = & C_0 + C_1 \cos \theta + C_2 \cos^2 \theta \\ & + \zeta \sin \theta \sin \varphi (D_0 + D_1 \cos \theta), \end{aligned} \quad (22)$$

where

$$C_0 = \frac{1}{2} (|a|^2 + |c|^2) p^2 k^2 + (|f|^2 + \frac{1}{2} |f + p^2 g|^2) |\xi|^2,$$

$$C_1 = pk \text{Re}[(a + b + c + p^2 d)^*(f + p^2 g) + 2b^* f] \xi,$$

$$\begin{aligned} C_2 = & p^2 k^2 [|b|^2 + \frac{1}{2} |b + p^2 d|^2 \\ & + \text{Re}(a^* c + (a + c)^*(b + p^2 d))], \end{aligned} \quad (23)$$

$$D_0 = pk \text{Im}[(a^* f - c^*(f + p^2 g)) \xi],$$

$$D_1 = p^2 k^2 \text{Im}(a^* b + a^* c + b^* c + p^2 d^* c).$$

We can now generalize the definition of the asymmetry  $A$  given in Ref. [6] to the polarized situation. Using the short-hand notation

$$\sigma(m; \theta, \varphi) \equiv \frac{d^2 \sigma}{dm^2 d\Omega}(\theta, \varphi), \quad (24)$$

we now define the angular-asymmetry parameter  $A$  through

$$\begin{aligned} A(m; \theta, \varphi) = & \frac{\sigma(m; \theta, \varphi) - \sigma(m; \pi - \theta, \varphi + \pi)}{\sigma(m; \theta, \varphi) + \sigma(m; \pi - \theta, \varphi + \pi)} \\ = & \frac{C_1 \cos \theta + \zeta D_0 \sin \theta \sin \varphi}{C_0 + C_2 \cos^2 \theta + \zeta D_1 \cos \theta \sin \theta \sin \varphi}. \end{aligned} \quad (25)$$

It follows from Eqs. (23) and (25) that  $C_1 = D_0 = 0$  for  $\xi = 0$  and thus

$$A(m; \theta, \varphi)_{\xi=0} = 0 \quad (26)$$

when isospin is conserved.

Let us discuss some specific features of the expression (25) in the following. First note that there are two different terms. The first term does not depend on the polarization  $\zeta$  of the proton beam and on the angle  $\varphi$ . Therefore, Eq. (25) implies that

$$A(m; \theta = 0^\circ, \varphi) = \frac{C_1}{C_0 + C_2}. \quad (27)$$

This particular result for the asymmetry  $A$  was derived earlier in Ref. [6] but not in terms of the amplitudes  $a, b, c, d, f$ , and  $g$ . The discussion in Refs. [7,22], on the other hand, takes into account only the amplitudes  $a, b, c$ , and  $f$ , but not the amplitudes  $d$  and  $g$ .

At  $\theta = 90^\circ$  and for  $\zeta \neq 0$  we get

$$\begin{aligned} A(m; \theta = 90^\circ, \varphi) = & \zeta \frac{D_0}{C_0} \sin \varphi \\ = & \frac{2\zeta pk \text{Im}[(a^* f - c^*(f + p^2 g)) \xi] \sin \varphi}{(|a|^2 + |c|^2) p^2 k^2 + (2|f|^2 + |f + p^2 g|^2) |\xi|^2}. \end{aligned} \quad (28)$$

Obviously the isospin-breaking effect for different angles  $\theta$  and  $\varphi$  depends on different combinations of the basic amplitudes. Thus, experimental information on the asymmetry obtained with polarized beam could allow us to deduce additional constraints on the amplitudes  $a, b, c, d, f$ , and  $g$  of the  $a_0$ - and  $f_0$ - production reactions. With regard to that let us mention that the model of  $a_0$ -meson production discussed in Ref. [6], which was based on the impulse approximation, leads to  $A(m; \theta = 90^\circ, \varphi) \equiv 0$ —because in that model  $a = c = 0$ .

Throughout this paper we make the assumption that isospin breaking takes place only in the propagation of the scalar mesons. This implies that the mixing strength is the same for all partial waves. On the other hand, if a significant mixing takes place also in the initial  $NN$  interaction or in the primary production amplitude  $\mathcal{M}_1$ , there is no reason to expect this mixing as being partial-wave independent. Thus the study of polarization observables allows us to examine this basic model assumption in a clean way.

In addition it will be very interesting to study the  $m$  dependence of  $A(m; \theta, \varphi)$  in detail. By construction  $A$  projects on the isospin breaking pieces of the amplitude. Therefore,

the  $m$  dependence of  $A$  gives direct access to the  $m$  dependence of  $\Sigma_{01}(m)$  and thus to the mixing mechanism.

## 2. Higher energies

As mentioned above the asymmetry  $A(m; \theta, \varphi)$  [Eq. (25)] will become zero if  $a_0^0$ - $f_0$  mixing is not included, i.e. for  $\xi = 0$ . This result is in line with the observation made in Ref. [24] that the asymmetry  $A(\theta)$  is identical to zero in the reaction  $pn \rightarrow d\pi^0$  for the unpolarized case. Let us emphasize, however, that the vanishing of the asymmetry  $A(m; \theta, \varphi)$  in the limit  $\xi = 0$  for the polarized case is true only near threshold, where the  $a_0$  meson is produced in a  $p$  wave and the  $f_0$  meson in an  $s$  wave, see Eqs. (18) and (19). The expressions (18) and (19) do not take into account contributions from higher partial waves, specifically they do not include, e.g., the  $d$  wave production of the  $a_0$  meson and the  $p$  wave production of the  $f_0$  meson. In the following we want to consider this question in more detail, assuming that isospin is conserved.

The amplitudes for  $a_0$ - and  $f_0$ -production in the reaction (1) can be written in the most general form as

$$\mathcal{M}_I = \phi_1^T \sigma_2 (F_I + \mathbf{G}_I \cdot \boldsymbol{\sigma}) \phi_2, \quad (29)$$

where  $\phi_{1,2}$  are the spinors of the nucleons. The two terms  $F_I$  and  $\mathbf{G}_I$  correspond to  $S_{NN}=0$  and  $S_{NN}=1$ , respectively, where  $S_{NN}$  is the total spin of the initial  $NN$  system. The subscript  $I$  denotes the isospin of the produced scalar meson. The scalar and vector functions  $F_I$  and  $\mathbf{G}_I$  depend on the vectors  $\mathbf{p}$ ,  $\mathbf{k}$  and  $\boldsymbol{\epsilon}$ . Let  $L_{NN}$  and  $L_{Md}$  be the angular momenta in the initial  $NN$  and in the final meson+ $d$  systems, respectively. It follows from the conservation of quantum numbers together with the required antisymmetry of the system with respect to the initial nucleons that the  $F_1$  ( $\mathbf{G}_1$ ) term in Eq. (29) should contain only the contributions from even (odd) values of  $L_{NN}$  and  $L_{Md}$ . One can also see that the  $F_0$  ( $\mathbf{G}_0$ ) term should only contain the contributions from odd (even) values of  $L_{NN}$  and  $L_{Md}$ . In the isovector ( $I=1$ ) case the  $s$  wave ( $L_{NN}=L_{Md}=0$ ) is forbidden, i.e., the contributions to the  $F_1$  term start with a  $d$  wave. For the case where one of the nucleons is polarized, the expression for the squared matrix element (29) is given by the form

$$\begin{aligned} |\overline{\mathcal{M}}_I|^2 = & \frac{1}{2} [|F_I|^2 + ([F_I \mathbf{G}_I^* + F_I^* \mathbf{G}_I] \cdot \boldsymbol{\zeta}) + (\mathbf{G}_I^* \cdot \mathbf{G}_I) \\ & + i(\boldsymbol{\zeta} \cdot [\mathbf{G}_I \times \mathbf{G}_I^*])], \end{aligned} \quad (30)$$

where  $\boldsymbol{\zeta}$  is the proton polarization vector. The second term in the expression (30) corresponds to the interference between the  $F_I$  and  $\mathbf{G}_I$  amplitudes. Thus, this term is an odd function of the final momentum  $\mathbf{k}$ . All other terms in Eq. (30) are even functions of  $\mathbf{k}$ . We see that the second term is responsible for the angular asymmetry in the reaction (1) even if isospin is conserved. It vanishes in the case of unpolarized protons, i.e.,  $A(m; \theta, \varphi) \equiv 0$  for  $\boldsymbol{\zeta} = 0$ .

Note that the amplitudes (18) and (19) are special cases of the general form Eq. (29) with  $F_I \equiv 0$ . Also, setting  $F_I \equiv 0$  in Eq. (30) we always get  $A(m; \theta, \varphi) \equiv 0$  if isospin is con-

served. The amplitudes (18) and (19) correspond to the  $\mathbf{G}_I$  terms in Eq. (30) in the lowest order approximation with respect to the final momentum  $\mathbf{k}$ , i.e., keeping only contributions that are at most linear in  $\mathbf{k}$ . The next order term with respect to  $\mathbf{k}$  in the amplitude for  $a_0$ -meson production is the  $d$ -wave contribution. This term (of the order  $\sim k^2$ ) contributes to  $F_1$  in Eq. (29) and modifies the amplitude  $\mathcal{M}_1$  given in Eq. (18), i.e.,

$$\mathcal{M}_1 \rightarrow \mathcal{M}_1 + F_1 \phi_1^T \sigma_2 \phi_2, \quad F_1 = e(\boldsymbol{\epsilon}^* \cdot [\mathbf{p} \times \mathbf{k}])(\mathbf{p} \cdot \mathbf{k}), \quad (31)$$

where  $e$  is a scalar amplitude. It is clear that the asymmetry  $A(m; \theta, \varphi)$  (25), calculated with the amplitude (31), will get nonzero contributions  $\sim k^3$  for small  $k$  due to the interference between  $p$  and  $d$  waves. It follows from Eq. (31) that  $F_1 = 0$  at  $\sin \theta = 0$  or  $\cos \theta = 0$ . In general, the  $F_1$  term contains all contributions with even values  $L_{NN}, L_{Md} = 2, 4, 6, \dots$ . Thus, each contribution should include the factor  $\boldsymbol{\epsilon}^* \cdot [\mathbf{p} \times \mathbf{k}]$  as well as the factor  $\mathbf{p} \cdot \mathbf{k}$ . Therefore, even in the general case  $F_1 = 0$  if  $\sin \theta = 0$  or  $\cos \theta = 0$ . Accordingly, for the case of conserved isospin we get

$$A(m; \theta = 0, \varphi) \equiv 0 \quad (32)$$

at any  $k$ —and not only near threshold [compare this result with that from Eq. (26)]. This result (32) confirms a general statement made in Ref. [24] on the absence of a forward-backward asymmetry for the reaction  $pn \rightarrow dX^0$  in the limit of isospin conservation.

It is remarkable that in addition to the condition (26) for the case of polarized proton beam we get also the nontrivial result that

$$A(m; \theta = 90^\circ, \varphi) \equiv 0 \quad (33)$$

if isospin is conserved. Therefore, any deviation of  $A(m; \theta = 90^\circ, \varphi)$  from zero is a direct indication for  $a_0^0$ - $f_0$  mixing. Note that this statement does not depend on the number of partial waves taken into account, i.e., it is valid for any excess energy and not only near threshold. Consequently, a measurement of the asymmetry at  $\theta = 90^\circ$  should provide us with evidence on the  $a_0^0$ - $f_0$  mixing amplitude. We want to remark that this information is to be considered as complementary to the one that can be obtained with unpolarized beam. It should be mentioned that the vanishing of the asymmetry at  $\theta = 90^\circ$  for the reaction  $pn \rightarrow da_0^0$  does not follow from the theorem formulated in Ref. [24] and is new.

Finally, let us discuss the situation at  $\theta \neq 0^\circ$  and  $\theta \neq 90^\circ$ . If the amplitude for  $a_0^0$ -meson production is taken of the form (20), i.e., isospin-breaking effects are included, then the leading order contribution to the asymmetry  $A(m; \theta, \varphi)$  is of the order  $\sim \xi k$  [see Eqs. (23) and (25)]. The next-to-leading terms in the amplitude  $\mathcal{M}_1$  [see Eq. (31)] give rise to contributions of the order  $\sim k^3$  to the asymmetry from isospin-conserving terms. Thus, when studying isospin-breaking effects over a wide region of  $\theta$ , one should consider  $a_0^0$ -meson production in a rather narrow region of relative momenta  $k$ ,

i.e., at small excess energies, in order to suppress contributions to  $A(m; \theta, \varphi)$  from isospin-conserving amplitudes with higher angular momenta.

## VI. SUMMARY

Let us summarize the main results of this paper. The most sensitive observables for examining the  $a_0^0$ - $f_0$  mixing amplitude are (i) the angular asymmetry  $A(m; \theta, \varphi)$ , as defined by Eq. (25) of this paper, and (ii) the distribution of the effective mass of the  $\pi^0\eta$  system near the  $a_0^0$  threshold. With regard to the angular asymmetry  $A(m; \theta, \varphi)$  for the channel  $pn \rightarrow d\pi^0\eta$ , it is expected to be rather large, i.e., in the order of  $\sim 10^{-1}$ . For the case of an unpolarized beam the forward-backward asymmetry was estimated earlier in Ref. [6]. The study of the asymmetry  $A(m; \theta, \varphi)$  with perpendicular polarized proton beam should shed additional light on the mechanism of  $a_0$  production in the reaction  $pn \rightarrow da_0^0$ . As we have shown, the various basic amplitudes for this reaction give different contributions to  $A(m; \theta, \varphi)$  at different angles.

Since by construction  $A$  projects on the isospin breaking pieces of the amplitude, the  $m$  dependence of  $A$  gives direct access to the  $m$  dependence of  $\Sigma_{01}(m)$  and thus to the mixing mechanism.

Note, in experiments with polarized beams, isospin-breaking effects give contributions  $\sim \xi k$  to the asymmetry  $A(m; \theta, \varphi)$  in leading-order with respect to the final momentum  $k$ . Isospin-conserving amplitudes give contributions  $\sim k^3$  due to  $d$ -wave terms. Thus when studying isospin-breaking effects over a wider range of  $\theta$  and  $\varphi$ , one should consider  $a_0^0$ -meson production at small momenta  $k$ , or small excess energies. Accordingly, when studying the reaction  $pn \rightarrow d\pi^0\eta$  near the threshold of  $a_0$  production, one should limit the invariant mass  $m$  of the  $\pi^0\eta$  system in a narrow region near threshold. Note that the contribution of the  $d$ -wave to the reaction  $pn \rightarrow da_0$  may be monitored by a simultaneous measurement of the differential cross section.

If isospin is conserved, then asymmetry effects should be absent at  $\theta=0^\circ$  or  $\theta=90^\circ$ , even in the polarized case. The angle  $\theta=0^\circ$  is not conclusive for the reaction with a polarized beam, since all polarization effects are proportional to  $\zeta \sin \theta \cos \varphi$  and vanish at  $\sin \theta=0$ . Thus, the asymmetry for  $\theta=0^\circ$  coincides with the one for unpolarized nucleons. The case  $\theta=90^\circ$  is much more interesting when studying the isospin-breaking effects in the reaction with polarized protons, since polarization effects are maximal at this angle. Anyway, in either case ( $\theta=0$  and  $\theta=90^\circ$ ) a nonvanishing asymmetry  $A(m; \theta, \varphi)$  can come only from isospin-breaking effects.

Very important information on  $a_0^0$ - $f_0$  mixing can be also expected from a study of the mass distribution  $d\sigma/dm_{\pi^0\eta}$  for the  $\pi^0\eta$  system in the reaction  $pn \rightarrow d\pi^0\eta$ . If the isospin is conserved, then the mass spectrum of the  $\pi^0\eta$  system from this reaction should coincide with the one for the  $\pi^+\eta$  system from the reaction  $pp \rightarrow d\pi^+\eta$ . Differences in the mass distributions of the  $\pi^0\eta$  and the  $\pi^+\eta$  systems are a clear signal that the isospin is not conserved and that  $a_0^0$ - $f_0$  mixing is present. The mass distribution  $d\sigma/dm_{\pi^0\eta}$  for the  $\pi^0\eta$  system is also extremely sensitive to the mechanism of  $a_0^0$ - $f_0$  mixing. For example, in the case where  $a_0^0$ - $f_0$  mixing is dominated by the  $K\bar{K}$  loop, the  $\pi^0\eta$  mass spectrum will be strongly enhanced near the  $K\bar{K}$  threshold.

Thus, we conclude that the study of the reaction  $pn \rightarrow da_0^0$  is extremely useful to understand the mechanism and the magnitude of the  $a_0^0$ - $f_0$  mixing amplitude. The knowledge of this mixing amplitude should allow us to shed light on the nature and the quark content of the lightest scalar mesons  $a_0(980)$  and  $f_0(980)$ .

Several interesting observables were not discussed in this paper and are to be studied in a subsequent work. One example is the differential cross section for  $pn \rightarrow dK^+K^-$ . Since this final state does not have definite isospin the differential cross section develops a forward-backward asymmetry at larger excess energies  $Q$ . Models predict that the kaon loops are an important source for the mixing [4]. Therefore, the measurement of a forward-backward asymmetry for the process  $pn \rightarrow dK\bar{K}$  (the sum of  $pn \rightarrow dK^+K^-$  and  $pn \rightarrow dK^0\bar{K}^0$ ) should help to pin down the  $a_0^0$ - $f_0$  mixing amplitude. Corresponding measurements of these decay channels should be feasible at the ANKE and TOF facilities, respectively, at the COSY accelerator in Jülich.

Finally, let us consider the reaction  $dd \rightarrow {}^4\text{He}\pi^0\eta$ . Evidently, here isospin is not conserved. Near the  $a_0$  threshold this reaction should proceed only through the chain  $dd \rightarrow {}^4\text{He}f_0 \rightarrow {}^4\text{He}a_0^0 \rightarrow {}^4\text{He}\pi^0\eta$ . Thus, the  $\pi^0\eta$  mass spectrum of this reaction should be even more sensitive to the  $a_0^0$ - $f_0$  mixing mechanism than the one for the reaction  $pn \rightarrow da_0^0$  [7].

## ACKNOWLEDGMENTS

The authors are thankful to L. A. Kondratyuk and M. Büscher for stimulating discussions. This work was partly supported by DFG-RFBI Grant No. 02-02-04001 (436 RUS 113/652/1-1) and by RFBR Grant No. 00-15-96562.

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- [1] N. N. Achasov, S. A. Devyanin, and G. N. Shestakov, Phys. Lett. **88B**, 367 (1979); Yad. Fiz. **33**, 1337 (1981) [Sov. J. Nucl. Phys. **33**, 715 (1981)].
  - [2] T. Barnes, Phys. Lett. **165B**, 434 (1985).
  - [3] N. N. Achasov and G. N. Shestakov, Phys. Rev. D **56**, 212 (1997).

- [4] O. Krehl, R. Rapp, and J. Speth, Phys. Lett. B **390**, 23 (1997).
- [5] B. Kerbikov and F. Tabakin, Phys. Rev. C **62**, 064601 (2000).
- [6] A. E. Kudryavtsev and V. E. Tarasov, JETP Lett. **72**, 401 (2000); nucl-th/0102053.
- [7] V. Yu. Grishina, L. A. Kondratyuk, M. Büscher, W. Cassing, and H. Ströher, Phys. Lett. B **521**, 217 (2001).



- [8] F. E. Close and A. Kirk, Phys. Lett. B **489**, 24 (2000).
- [9] D. Barberis *et al.*, Phys. Lett. B **488**, 225 (2000).
- [10] A. Kirk, Phys. Lett. B **489**, 29 (2000).
- [11] G. Janssen, B. Pierce, K. Holinde, and J. Speth, Phys. Rev. D **52**, 2690 (1995).
- [12] J. A. Oller and E. Oset, Nucl. Phys. **A620**, 438 (1997); **A652**, 407 (1999).
- [13] S. Weinberg, Phys. Rev. **130**, 776 (1963); **131**, 400 (1963).
- [14] E. Oset, J. A. Oller, and Ulf-G. Meissner, Eur. Phys. J. A **12**, 435 (2001).
- [15] S. M. Flatté, Phys. Lett. **63B**, 224 (1976).
- [16] N. N. Achasov, S. A. Devyanin, and G. N. Shestakov, Yad. Fiz. **32**, 1098 (1980) [Sov. J. Nucl. Phys. **32**, 566 (1980)].
- [17] W. B. Tippens *et al.*, Phys. Rev. D **63**, 052001 (2001).
- [18] Particle Data Group, D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).
- [19] V. Yu. Grishina, L. A. Kondratyuk, E. L. Bratkovskaya, M. Büscher, and W. Cassing, Eur. Phys. J. A **9**, 277 (2000).
- [20] C. Hanhart and K. Nakayama, Phys. Lett. B **454**, 176 (1999).
- [21] R. A. Arndt, I. I. Strakovsky, and R. L. Workman, Phys. Rev. C **62**, 034005 (2000).
- [22] M. Büscher *et al.*, Investigation of Neutral Scalar Mesons  $a_0/f_0$  with ANKE, COSY Proposal No. 97, 2001, <http://ikpd15.ikp.kfa-juelich.de:8085/doc/Proposals.html>
- [23] S. Teige *et al.*, Phys. Rev. D **59**, 012001 (1999).
- [24] G. A. Miller, B. M. K. Nefkens, and I. Šlaus, Phys. Rep. **194**, 1 (1990).